Central Limit Theorem

- Sampling Distribution
- Central Limit Theorem
- Normal Approximation to the Binomial

Stock Portfolios

- http://money.cnn.com/

Sampling Distributions

- Sampling Distribution: Given a population and sample size n, the sampling distribution of a statistic is the distribution of all values of the statistic for all possible samples of size n.

Sampling Distribution of the Mean

- Sampling Distribution of the Mean: Given a population and sample size n, the sampling distribution of the mean is the distribution all the means for all possible samples of size n.

Sampling Distribution of the Proportion

- Sampling Distribution of the Proportion: Given a population and sample size n, the sampling distribution of the proportion is the distribution all the proportions for all possible samples of size n.

Sampling Distributions

- The Mean of the Sample Means: $\mu_{\bar{x}}$
- The Standard Deviation of the Sample Means $\sigma_{\bar{x}}$
- The Mean of the Sample Proportions: $\mu_{\hat{p}}$
- The Standard Deviation of the Sample Proportions: $\sigma_{\hat{p}}$

Biased and Unbiased Estimator

Unbiased estimator: A statistic whose sampling distribution has a mean that is equal to the population parameter.

- The sample mean is an **unbiased estimator** for the population mean.
- The sample proportion is an **unbiased estimator** for the population proportion.
- The sample variance is an **unbiased estimator** for the population variance.
- The sample median is a **biased estimator** for the population median.

Simulations

- http://www.socr.ucla.edu/Applets.dir/SamplingDistributionApplet.html

The Central Limit Theorem

Let $\bar{x}$ denote the mean of a random sample of size n from a population having mean μ and standard deviation σ. Let

$\mu_{\bar{x}}$ = the mean of the $\bar{x}$ distribution

$\sigma_{\bar{x}}$ = the standard deviation of the $\bar{x}$ distribution

then

$$\mu_{\bar{x}} = \mu \quad \text{and} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

When the distribution is normal so is the distribution of $\bar{x}$ for any n.

For large n, the distribution of $\bar{x}$ is approximately normal regardless of the population distribution.

Rule of Thumb: $n > 30$ is large

CLT Example

Suppose the average amount of time that it takes for college students to complete their degree is 4.7 years. The standard deviation is 0.3. What is the probability that 40 randomly selected college students will have a average completion time of less than 4.4 years?

CLT Example

Suppose stock increases are normally distributed with a mean of 3 percent and a standard deviation of 5 percent. If your (randomly selected portfolio) consists of 20 stocks, what is the probability that your portfolio will lose money?

CLT Example

Suppose the average GPA of college students is 3.1 and the standard deviation is 0.7. A class of 35 randomly selected students will be considered high risk of their mean GPA is in the bottom 2nd percentile. What is the largest mean GPA that will be considered high risk?

The CLT for Proportions

- Requirements: Must be a Binomial Distribution with $np > 5$, $nq > 5$ ($q = 1\text{-}p$)
- Conclusion: This Binomial Distribution is approximately normal with

$$\mu = np, \quad \sigma = \sqrt{npq}$$

- Continuity Correction: Adjust the discrete whole number x by 0.5.

Continuity Correction

Binomial	Normal
P(x < 10)	P(x < 9.5)
P($x \leq$ 10)	P(x < 10.5)
P(x > 10)	P(x > 10.5)
P($x \geq$ 10)	P(x > 9.5)
P(7 < x < 12)	P(7.5 < x < 11.5)
P(7 $\leq x \leq$ 12)	P(6.5 < x < 12.5)

Example

Twelve percent of the US population is left handed. If 200 randomly selected Americans are surveyed, what is the probability that fewer than 20 of them are left handed?

Example

According to a recent Gallup poll, 18% of Americans are underemployed. If 150 Americans are randomly selected, find the probability that between 20 and 30 of them are underemployed.

Testing a Claim

Is it likely that only 50% of voters support the marijuana initiative? A recent field poll of 1000 decisive voters found that 547 of them will vote yes.