Confidence Interval for a Population Proportion

- Confidence Interval Computation
- Confidence Interval Interpretation
- With the TI 83/84
- Determining the Sample Size

Current Event

http://www.gallup.com/poll/143960/Election-Measures-Continue-Predict-Major-House-Shakeup.aspx

Confidence Interval Terminology

- Point Estimate: The “best” estimate for the population parameter using the sample data.
- Standard Error: The standard deviation of the sampling distribution
- Margin of Error: The maximum likely difference between the observed statistic and the population parameter.
- Confidence Level: The probability $1-\alpha$ that the confidence interval will contain the population mean.
- Critical Value: The value of z such that the area of the normal curve between $-z$ and z is $1-\alpha$.

Confidence Interval Definition

A $1-\alpha$ Confidence Interval for the population parameter is an interval centered about the sample statistic with width equal to twice the margin of error. If many samples are taken from a population with the same sample size then the proportion of the constructed confidence intervals that will contain the population parameter is $1-\alpha$.

Confidence Interval Example

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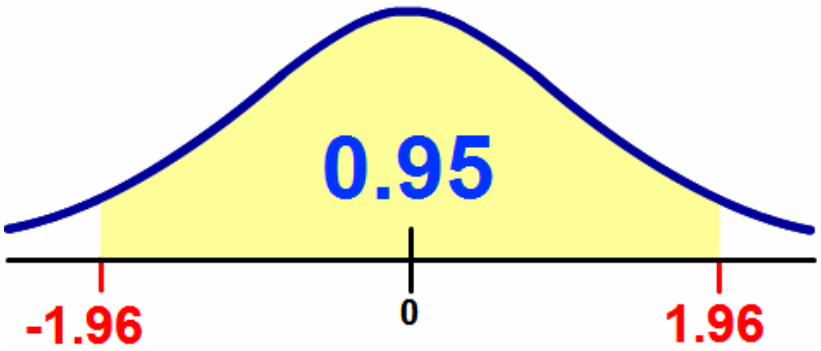
$$n = 935$$

$$\hat{p} = \frac{312}{935} \approx 0.33$$

$$np, nq > 5$$



$$\mu_{\hat{p}} \approx \hat{p} = 0.33$$

$$z = \frac{x - \mu}{\sigma / \sqrt{n}} \qquad x = \mu + z\frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\hat{p}} \approx \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.33 \bullet 0.67}{935}} \approx 0.015$$

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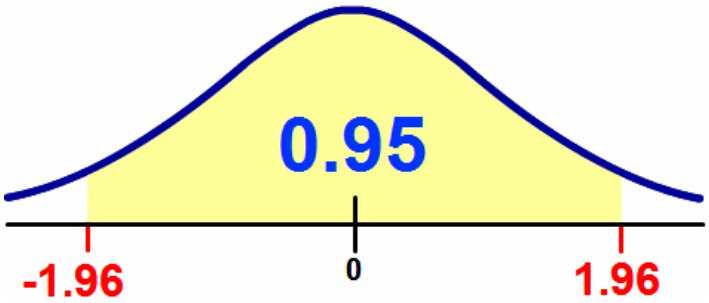

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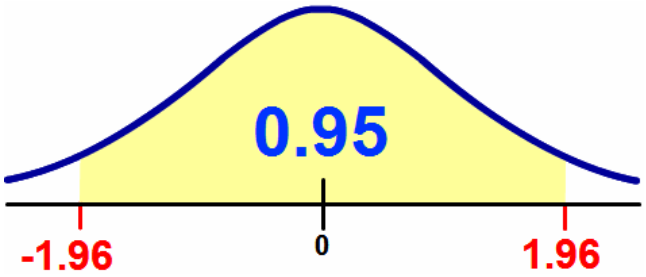

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If we took many, many polls each with sample size 935 then each of these polls would result in different confidence intervals. 95% of all of these confidence intervals will contain the true population proportion for all registered voters.

Using the TI 83/84

1. STAT
2. TESTS
3. 1-PropZInt (scroll down far)
4. ENTER
5. x = number of successes
6. n = sample size
7. C-Level = Confidence Level (1-α)
8. Calculate

Example

A study was done to estimate the proportion of online college students who feel like they are enrolled in too many classes. Of the 150 students who were surveyed, 60 of them answered that they were. Determine a 95% confidence interval.

STAT → TESTS → 1-PropZInt

Example

A biologist wants to estimate the proportion of Tahoe Chickaree squirrels that survive the winter. The biologist tagged 450 randomly selected squirrels in the fall. By spring, only 320 of them were still alive. Determine and interpret the 90% confidence interval for the population proportion.

STAT → TESTS → 1-PropZInt

Determining the Sample Size

No preliminary estimate for p: $n = \dfrac{0.25 \bullet \left(z_{\alpha/2} \right)^2}{E^2}$

With preliminary estimate for p: $n = \dfrac{pq \bullet \left(z_{\alpha/2} \right)^2}{E^2}$

Example

You want to perform a study to estimate the proportion of college students who receive financial aid. You want to construct a 95% confidence interval with a margin of error no more that ±6%.

A. How many people should you survey if you have no idea in advance what the proportion is?

B. Last year, the college found that 30% of all college students receive financial aid. How many people should you survey this year?

No preliminary estimate for p: $n = \dfrac{0.25 \cdot (z_{\alpha/2})^2}{E^2}$

With preliminary estimate for p: $n = \dfrac{pq \cdot (z_{\alpha/2})^2}{E^2}$