Confidence Interval for a Population Mean

- Estimating μ (σ known)
- Estimating μ (σ unknown)
- Sample Size

Estimating μ (σ known)

$$u_{\bar{x}} = \mu \approx \bar{x}, \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$CI : \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Effects on Confidence Intervals

$$CI: \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

σ: Decreasing σ decreases the margin of error and the width of the confidence interval.

n: Increasing n decreases the margin of error and the width of the confidence interval.

$1-\alpha$: Decreasing $1-\alpha$ (95% goes to 90% confidence) decreases the critical value and thus the margin of error and the width of the confidence interval.

Example

Every year UC Davis measures the clarity of the lake. Based on historical data the standard deviation of clarity throughout the year is 1.6 meters. Suppose that in 2010, the Davis researcher took 35 measurements and found the mean depth of clarity to be 21.7 meters. Find a 95% confidence interval for the average depth of clarity of the lake.

STAT → TESTS → ZInterval → Stats

Example

A restaurant owner wants to estimate the mean amount of money her customers spend at her restaurant. She knows that the standard deviation is \$6.42. She looks at 50 randomly selected receipts and calculates the mean of these receipts to be \$43.71. Determine and interpret the 90% confidence interval for the mean.

Example

Among all Americans the standard deviation for amount of time spent watching TV per week was 4 hours. A study was done to estimate the mean number of viewing hours per week this year. The 45 randomly selected people surveyed averaged 18 hour per week. Determine and interpret the 95% confidence interval for the mean.

Confidence Intervals (σ Unknown)

Since the z-score formula involves the population standard deviation, we cannot use z for a confidence interval unless σ is known. Instead, we use a distribution called the Student's t-Distribution. This distribution has an additional parameter, called the degrees of freedom, which is n-1. The rest of the confidence interval calculation is the same as before, but we use the sample standard deviation s as a point estimate for the population standard deviation. To use this you must either have an approximately normal distribution or a sample size larger than 30.

Example: CI for σ Unknown

A study was done to estimate the mean age when people buy their first new car. The mean age of purchase for the 32 randomly selected people was 22 and the standard deviation was 3. Determine and interpret the 95% confidence interval for the mean.

STAT → TESTS → TInterval → Stats

$$CI: \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

Example: CI for σ Unknown

A study was done to estimate the mean blood alcohol level for customers at the casino tables. The 25 randomly selected table players had a mean level of 0.09 and the standard deviation was 0.024. Determine and interpret the 95% confidence interval for the mean. Assume that the distribution of table player's blood alcohol level is approximately normally distributed.

STAT → TESTS → TInterval → Stats

Determining the Sample Size

Suppose that you want to conduct a study so that you can construct a 90% confidence interval for the mean number of physical therapy visitations a patient needs after receiving ACL surgery. You want the margin of error to be no more than 0.5 visits. If you know the standard deviation is 4 visits, how many patients must participate in this study?

$$E = z_c \frac{\sigma}{\sqrt{n}}$$

$$n = \left(\frac{z_c \sigma}{E} \right)^2$$